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Review: Fatigue-crack propagation in metallic and polymeric materials

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Fatigue-crack propagation in metals and polymers is examined from the point of view of both the materials scientist and the engineer. Progress over the last few years is outlined, and possible future developments suggested.

1. Introduction

Fatigue, the phenomenon in which a crack develops and extends under cyclic stressing conditions, accounts for the vast majority of service failures. All types of materials are liable to fatigue fracture, but present-day knowledge is largely associated with crystalline solids, particularly metals and alloys. Consequently, the subject has been studied in depth by both engineers and metallurgists; the former being concerned with the endurance or lifetime of components under given fluctuating stresses, and the latter investigating the structural changes involved. Different languages are employed by the two disciplines and it was evident at a recent meeting [1] that substantial communication problems still exist. This article will present a general picture of fatigue-crack growth, and also attempt where possible to draw together some of the interdisciplinary threads. Since cyclic crack growth in polymeric materials has received only limited attention to date [2, 14], each section of the review will first consider metallic behaviour as a baseline, and subsequently compare and contrast the response of polymers if this is known.

In crystalline materials the fatigue process is commonly subdivided into three events prior to fracture; initiation, stage I crack growth and stage II crack growth [3]. The last of these, where the crack has macroscopic dimensions, is probably of major importance to the engineer, and has been most studied by the metallurgist/materials scientist. However, before discussing the actual growth of a fatigue crack it is pertinent to examine the circumstances surrounding its initiation.

2. Initiation and stage I growth

The single application of a stress, in excess of the local yield value, to a crystalline material results in the production and movement of dislocations, which form slip bands or slip lines where they intersect the surface of the specimen. On repeated stressing the subsequent deformation is concentrated into certain of these slip steps, which become broader and deeper until eventually a micro-crack initiates. Incomplete reversibility of slip accounts for this process, but the precise dislocation mechanisms involved have not yet been unequivocally elucidated. Very thin folds of material known as *extrusions* are pushed out of the surface, and they are generally associated with small crevices, termed *intrusions* penetrating along the slip planes of maximum resolved shear stress. A stage I fatigue crack is simply an enlarged intrusion, and so any demarcation between initiation and stage I growth would appear unrealistic. The intrusion-extrusion mechanism may perhaps be regarded as the "classical" process of fatigue-crack initiation, although there are several other common sites at which cracks develop. These include interfaces of all types, and grain boundaries particularly at high strain amplitudes and/or elevated temperatures [4-6].

While intrusions and extrusions are being created at the surface, changes are also taking place within the interior of the specimen. A stable dislocation arrangement is set up, which is largely influenced by the cyclic stress amplitude and the stacking-fault energy of the material. A high value of the latter favours a continuous subgrain structure, whereas in materials in which cross slip is difficult, islands of very high dislocation density occur in an essentially dislocation free matrix. A recent review [7] has discussed at length these and other structural changes induced during fatigue-crack initiation. During this period also, significant alterations in mechanical properties may take place, which are of particular importance to the design engineer, who may employ the following empirical criterion to ascertain whether the strength has increased or diminished during cycling [8]. If the ratio of U.T.S.:0.2% P.S. exceeds 1.4, hardening occurs but if this ratio is less than 1.2 cyclic softening will result.

As the intrusion or stage I crack extends along the active slip plane further into the body of the specimen, shear growth becomes more difficult due to the increasing tensile stress component. Eventually the constraint is sufficient to cause a gradual macroscopic rotation [9] of the crack to a non-crystallographic plane approximately normal to the direction of the applied stress, and this marks the onset of stage II propagation (Fig. 1). The proportion of the total fatigue life spent in each stage is largely controlled by the stress amplitude. For small values of this the majority of the life (up to 90%) is taken up by the initiation and stage I growth processes, whereas at high cyclic stress levels stage II propagation may occupy a similar portion of the life. The existence of stress/strain concentrating features such as sharp corners, notches and large intermetallic particles tends to inhibit or even eliminate the initiation and stage I aspects of fatigue-crack growth. This fact reinforces the earlier statement that in real engineering components in which some stress concentrating

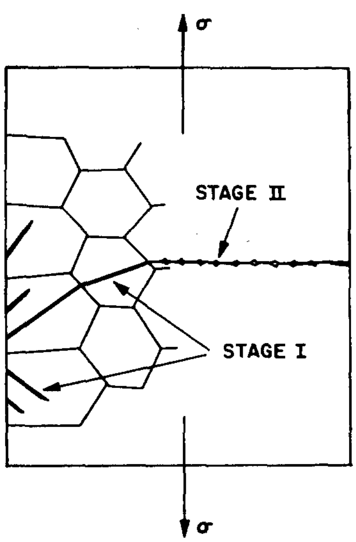


Figure 1 Schematic representation of the stages of fatigue-crack growth.

feature can normally be found, stage II crack propagation is the most significant aspect of fatigue failure.

Both the metallography and the factors influencing crack initiation and stage I propagation have been discussed in detail by Plumbridge and Ryder [7], and will not be listed here. Most of the variables affecting stage II growth also influence stage I and initiation to some degree, and these will be mentioned later.

An important difference in terminology must be emphasized at this point. It is quite common in engineering parlance to consider a crack which has grown to some arbitrary length, e.g. 0.050 in. (1.27 mm) or 0.125 in. (3.175 mm) as having "initiated", and sometimes this is also taken as a criterion of failure for design purposes. However, this "initiated" crack would be regarded as a stage II crack by the materials scientist, except in certain circumstances such as torsional fatigue where stage I cracks propagate over large distances along the specimen surface.

Because of a high hysteresis, the mechanical behaviour of polymers may be regarded as being considerably more dependent upon time and temperature than that of metallic materials. An outcome of this in terms of their response to cyclic stressing, is that three distinct categories of failure exist [14]. A cyclically induced "creep" failure may occur; a "thermal" failure may arise from structural changes caused by an increase in temperature; or under less severe conditions fracture may be a consequence of a process of crack initiation and growth. Only the last is relevant to this article. The boundary conditions for fatigue to occur by a crack nucleation and growth process in thermoplastic materials have been calculated by Constable *et al* [10], and within this range considerable cyclic softening still occurs. Cracks may initiate at the specimen surface or internally from existing flaws, the latter being favoured by high strain amplitudes [10-12]. In crystalline polymers a form of slip may possibly produce fresh initiation sites [13], while these may result from void formation during viscous flow in amorphous material. There appears to be no counterpart to stage I shear growth. Andrews [14] has reviewed the fatigue process in polymers, and provides a further definition of an initiated crack as being "that capable of growth under the applied constraints". He estimates the importance of initiation, as denoted by the fraction of the lifetime devoted to it, to be negligible in rubbers or rubbery plastics but considerable in crystalline polymers and in fibre-reinforced plastics where breakdown of the fibre-matrix interface may be regarded as initiation.

3. Stage II fatigue-crack propagation

This aspect of crack growth has received by far the most attention both qualitatively and quantitatively; the impetus being generated largely as a result of the Comet disasters of the mid-fifties. The following section will consider initially the qualitative information that has been provided by material scientists, and then examine the engineering attempts at measuring and predicting fatigue-crack growth rates.

3.1. Qualitative approach

During the early cycles following rotation of the crack to a plane of maximum tensile stress a small plastically deformed region is created around the crack tip, and as the crack extends, this zone becomes larger. Its shape has been approximated to two truncated cones meeting at mid-section [15] (Fig. 2), and when its dimension attains a critical fraction of the specimen thickness a further gradual rotation of the crack plane occurs to 45° to both the tensile axis and the section [16] (Fig. 3). Propagation prior to rotation is often termed "plane strain" (since the plastic zone size is relatively small) or "square" growth, and the crack extension on shear planes following rotation is referred to as "plane stress" or "slant" growth. Subsequent propagation takes place on the slant plane or a conjugate plane until final failure. Naturally, "all-slant" growth is most noticeable in sheet and plate specimens, in which the plastic zone and the thickness dimensions are often comparable. The path that the stage II crack takes is simply that which offers least resistance, and although a transgranular non-crystallographic route is most common, large fractions of intercrystalline fracture may occur when grain boundaries are weak and favourably oriented [17, 18], and this is particularly so in body centred cubic metals and alloys. The crack may also adhere to slip planes [15], follow quench bands [19], or be attracted by large internal stress concentrators [20]. It is not unusual for several of these modes to be active either simultaneously or sequentially during the propagation period.

Examination of stage II fatigue fracture surfaces has been extensive, employing optical, electron and scanning electron microscopy

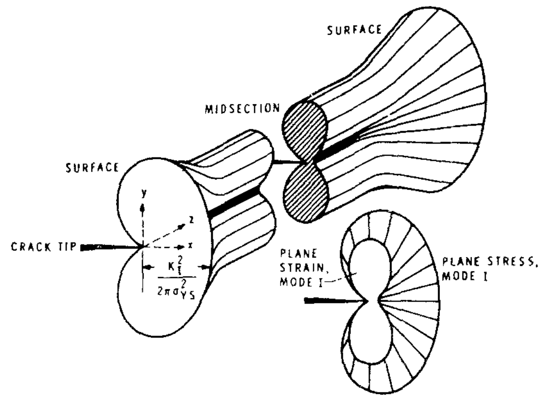


Figure 2 Simplified model of the crack-tip plastic-zone in a sheet or plate specimen.

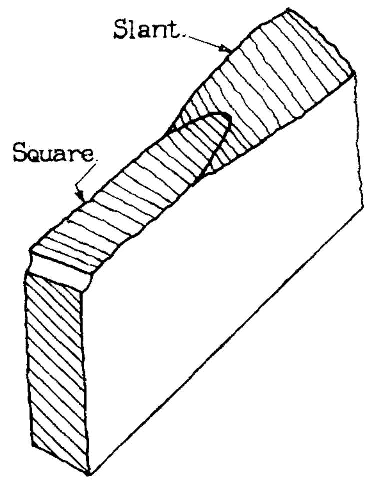


Figure 3 The square-to-slant growth transition of a crack initiating from a central notch. (Diagrammatic.)

techniques. For many years it was widely believed that the concentric concoidal markings known as "striations" [21] which lay approximately perpendicular to the local direction of crack growth, were a necessary requisite for the diagnosis of a fatigue failure. However, striations are usually found only on the square region of the stage II fracture surface, and it has been suggested that they form solely under conditions at the crack front which approximate to plane strain [22]. The distance between adjacent striations represents the local crack extension during one stress cycle, but the converse that each cycle produces a striation is not necessarily valid, especially in long-life fatigue where there may be redundant cycles. There are two varieties of fatigue striation designated "ductile" or type A and "brittle" or type B [23]. The latter, which are characterized by "river marks", are less common, but have been observed in certain fully hardened aluminium alloys particularly in corrosive environments [24], cold worked nickel alloys [25] and hydrogen embrittled iron [26]. On the slant portion of a stage II fracture surface, the "characteristic" striations are rarely found; a fact of great importance in service failure diagnosis. Electron fractographs of this region reveal several features, the most important being ductile dimples elongated perpendicular to the direction of macroscopic crack growth, and more commonly associated with tensile shear fracture. Smooth areas produced by attrition of the upper and lower fracture surfaces are also formed, together with particles of the resultant fretting product. There appears to be no correlation between dimple size and crack-growth rate, as was found for the striation spacing [27, 28]. Fig. 4 shows some typical examples of fatigue fracture surface topography. Several other secondary fractographic aspects are well described by Beacham [29]. Recently [30] the square region of stage II propagation in low carbon steel has been further subdivided according to whether growth is structure sensitive (stage IIa) or insensitive (stage IIb). The transition is said to occur at the disappearance of intergranular fracture, and this corresponds to a critical plastic zone: grain size ratio of about four [31]. It is unlikely that this represents a fundamental variation in stage II growth, particularly as the rate of crack propagation remains unaltered. (See section on effect of materials structure.)

It was probably somewhat unfortunate that the materials which were first extensively examined fractographically were the commercial aluminium alloys, because these exhibit clear and well defined fatigue striations. Their visibility in other alloys, particularly steels, is not so good. In thin sheet aluminium specimens cycled in fluctuating tension at low amplitudes they are absent, but appear in abundance when the same material is fatigued in reverse plane bending [22]. Striations tend to be less curved in low stacking-fault energy materials [32, 33]. Environment may also influence the fracture surface topography, as typified by the featureless regions (Fig. 5) produced on testing aluminium alloys in vacuum [34]. This has been attributed to rewelding of the fracture surfaces in the absence of an oxide film. Striation visibility may also be impaired by post-crack deformation of the fracture surface, especially in low yield strength materials, or the observational conditions may be unfavourable for their resolution. Broek [35] has clearly demonstrated the need for high-tilt facilities when viewing replicas of fracture

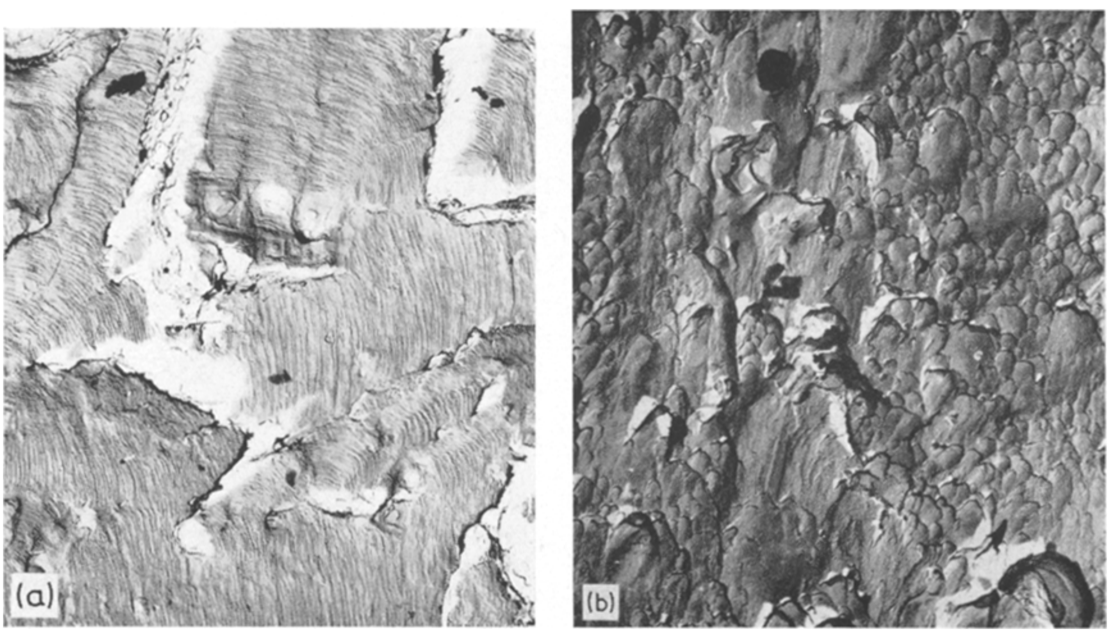
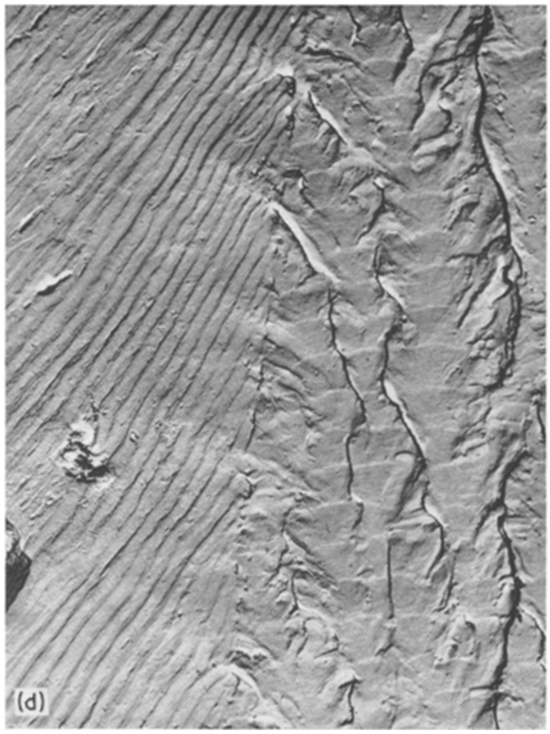
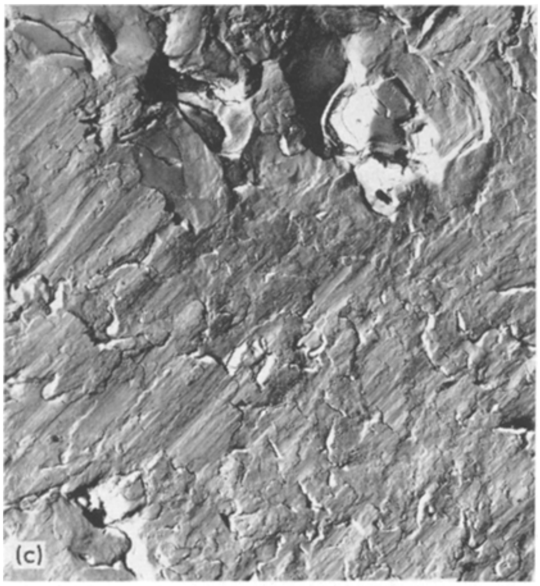


Figure 4





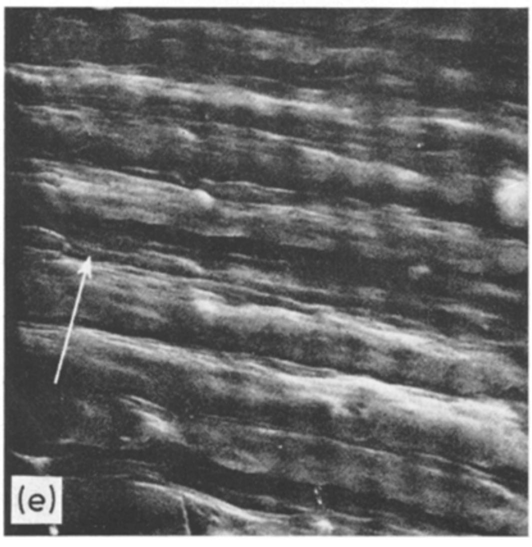


Figure 4 (a) Fatigue striations on the plane strain region of the fracture surface of a heat-treated duralumin alloy. Note wide variation in local direction of growth. (Two stage carbon-plastic replica, germanium shadowed, magnification × 2500 approx.) [28].

(b) Ductile dimples in fractograph from slant portion of the fracture surface of the same specimen. Magnification × 4000 approx. [28].

(c) Rub marks produced by attrition of upper and lower slant fracture surfaces. Magnification × 4000 approx. [28].

(d) Type A (L.H.S.) and type B fatigue striations on adjacent grains in an aluminium-zinc-magnesium alloy tested in distilled water. Magnification × 2500 approx. [119]. (Courtesy Chapman and Hall.)

(e) Scanning electron fractograph of ductile striations formed in aluminium during high strain cycling. $N_f = 460$ cycles. Magnification × 1650 approx. [12]. (Courtesy Chapman and Hall.)

surfaces in the electron microscope (Fig. 6).

Fatigue striations have been observed on the fracture surfaces of most polymeric materials examined to date, at temperatures both above and below the glass transition temperature [12, 36-39] (Fig. 7). At high strain amplitudes tearing may occur and break up the ripple pattern [12]. In natural rubber, as in metals, growth may proceed in several plateaux simultaneously, but unlike metals, this "rough" growth is associated with a dramatic reduction in propagation rate with respect to that when growth occurs on a single through thickness plane [36, 38]. The deviation of the maximum stress trajectory at the crack tip from the crack plane may result in striations having a cusped

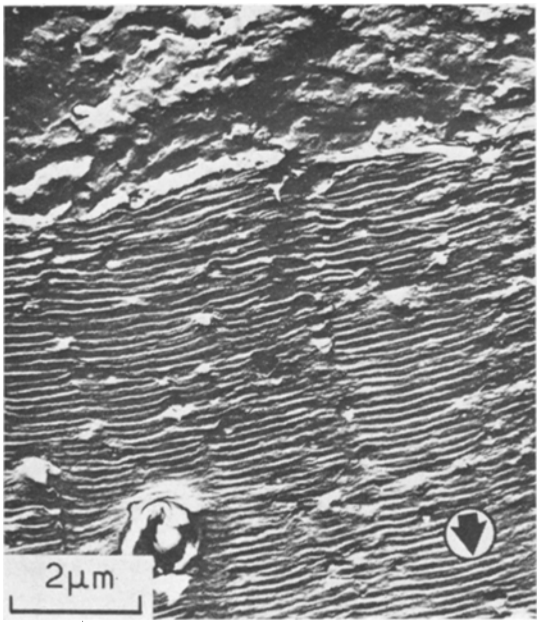


Figure 5 Fatigue-crack growth of an aluminium-copper alloy in vacuum, followed by growth in air. Carbon replica illustrating the effect on striation visibility [34]. (Courtesy American Society for Metals.)

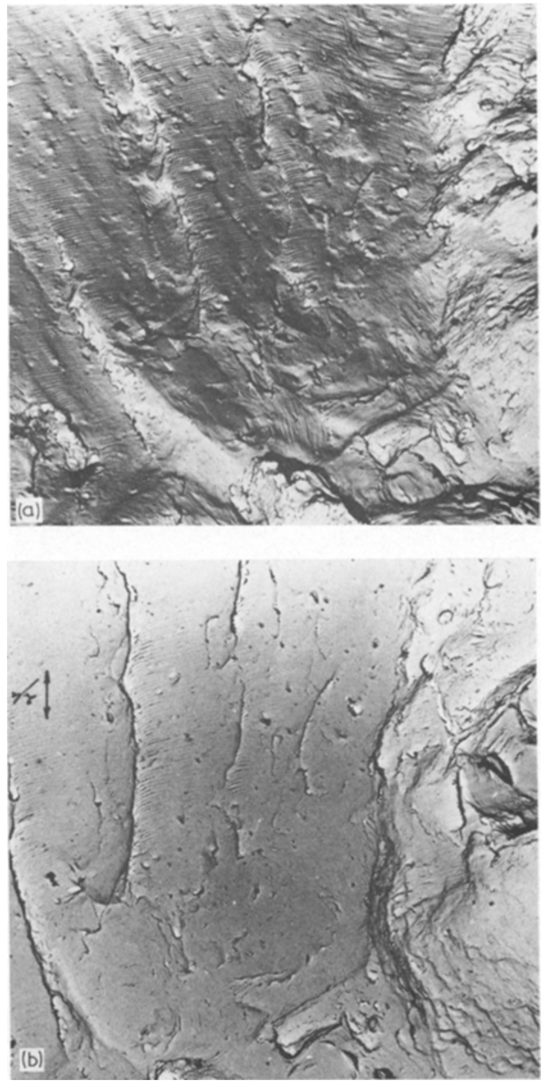


Figure 6 Effect of tilting on the visibility of striations in an aluminium-copper alloy. Tilt axis diagonal from lower left to upper right. (a) 45° tilt, (b) 0° tilt [35]. (Courtesy Pergamon Press.)

profile. Occasionally when alternate segments turn from the axis in opposite directions a "quilt-like" effect is observed [40].

It might be expected that fractography would provide some assistance in understanding "load-sequence" effects apparent in cumulative damage studies of fatigue at varying stress levels. For example, the Miner equation [41] assumes that damage accumulates linearly, i.e.

$$\sum \frac{n}{N_f} = 1$$

where n is the number of cycles applied at a particular stress level, and N_f the number of cycles to failure at the same level. Experimentally it is often found, however, that if low stresses are followed by high stresses the summation exceeds unity, but when the large stresses are applied first the summation is less than unity, which is a potentially dangerous situation as far as design is concerned. Although failure was originally defined as the appearance of a visible crack, several subsequent treatments have incorporated complete rupture as their failure criterion. It is therefore pertinent to enquire whether the load-sequence effect manifests itself in stage II propagation as well as in crack initiation. Unfortunately, the fractographic evidence to date is fairly inconclusive, with some findings [42] even suggesting a reverse load-sequence effect, in which residual compressive stresses induced by the high stress application retarded subsequent crack growth. Paris [43] was unable to detect any sequence effect under random loading. However, a careful study by Hertzberg [27] on an aluminium alloy does reveal a load-sequence effect in stage II propagation.

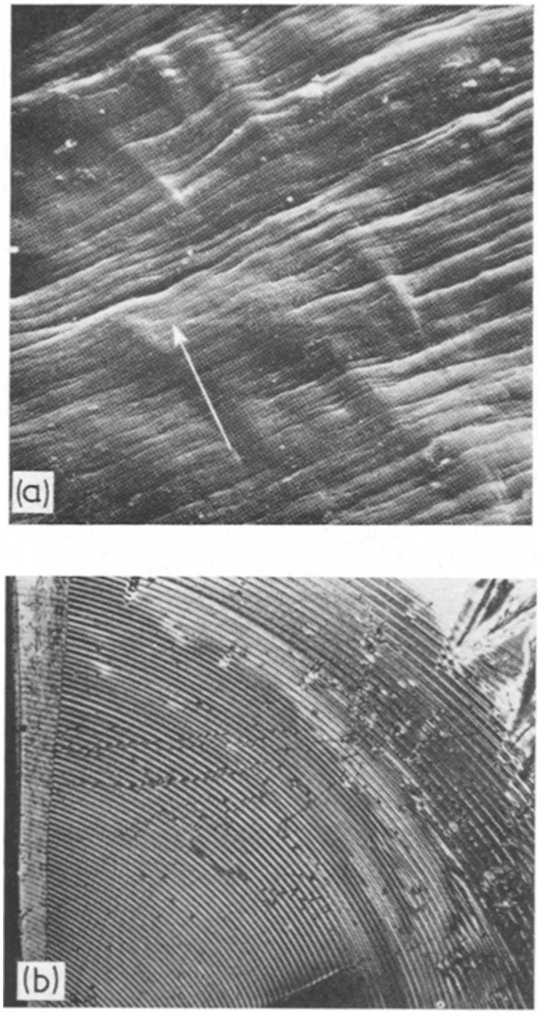


Figure 7 (a) Scanning electron micrograph of striations in fatigued nylon. Magnification × 350 approx. [12]. (Courtesy Chapman and Hall.)

(b) Optical fractograph of striations in fatigued polycarbonate. Magnification × 70 approx. [37]. (Courtesy Syracuse University Press.)

Due to damage accumulation, the spacings between the first few striations, produced at a low stress level following high stress cycles, were greater than subsequent low stress striation spacings. McMillan and Pelloux [44] provide confirmatory evidence, again for aluminium alloys, and demonstrate that the sequence effect is apparent only when the maximum stress is changed.

3.1.1. Mechanisms of crack growth

Considerable effort has been devoted to elucidating the mechanisms of stage II fatigue-crack propagation and striation formation. A general outline of a selection of these will now be considered (Fig. 8). Originally, Forsyth and Ryder [45], studying precipitation hardened aluminium alloys, suggested that cleavage fracture occurred ahead of the crack tip, and that the striation profile was formed by subsequent necking of the intermediate material. Although cleavage of brittle precipitates ahead of crack may well occur, this would result in the striation spacing being governed mainly by the inter-particle distance rather than the stress amplitude and the crack length. A more generalized approach has been made by Laird and Smith [46] after examining high strain crack growth in aluminium and nickel. They describe the growth process as one involving successive crack blunting on the tensile part of the cycle and re-sharpening during the compressive stroke. This plastic blunting process has been recently considered for crystalline materials on an atomic scale [47], and it has been suggested that when the dislocation density at the crack tip exceeds a critical value, a sub-grain forms which is able to absorb deformation in its wall and so arrest the crack. The increment of growth per cycle is therefore one sub-grain diameter. However, there is substantial evidence that the growth per cycle as indicated by striation spacing increases with increasing stress amplitude, while the sub-grain size in the bulk of the material decreases to a limiting value of about 2 μm [48]. Furthermore, transmission electron microscopy of subcutaneous regions of stage II fracture surfaces in copper reveals that there are several sub-grains between adjacent striations [9]. A third model for the mechanism of striation formation may loosely be termed the "crystallographic model" in which one or both sides of the striations are parallel to operative slip systems, and in which the positions of the slip planes determine the local angle of the striations with respect to the macroscopic growth direction [27, 44, 49]. Crack extension is restricted to the loading cycle only. A similar but more generalized approach is involved in the "sliding-off" or "shear decohesion" models in which separation occurs on alternating planes of maximum-resolved shear stress [50].

Because they are the least rigorous, and independent of structure, it is probable that the plastic blunting and the shear-decohesion models are the most universally applicable to stage II fatigue-crack growth in all materials. Neverthe-

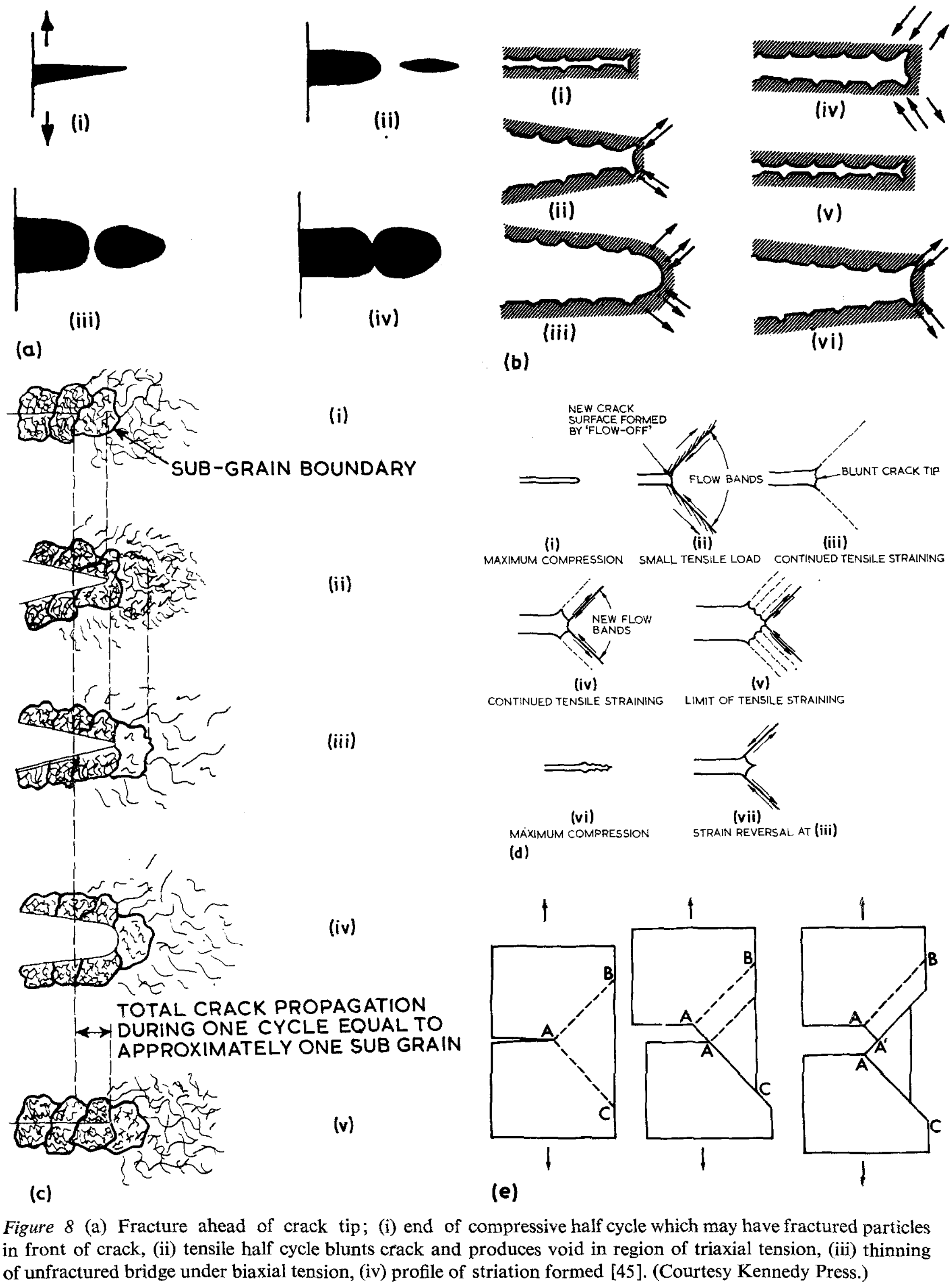


Figure 8 (a) Fracture ahead of crack tip; (i) end of compressive half cycle which may have fractured particles in front of crack, (ii) tensile half cycle blunts crack and produces void in region of triaxial tension, (iii) thinning of unfractured bridge under biaxial tension, (iv) profile of striation formed [45]. (Courtesy Kennedy Press.)

(b) Plastic blunting model of fatigue-crack propagation as load varies during one cycle. Double arrowheads signify increased width of slip bands [139]. (Courtesy American Society for Testing and Materials.)

(c) Dislocation model for plastic blunting. (i) critical dislocation density is reached (ii) and on re-arrangement a sub-grain is formed (iii), which limits growth in one cycle [47]. (Courtesy N.L.R. Amsterdam.)

(d) A typical model for "shear plane decohesion" or "sliding-off" on planes of maximum stress. Note that in the crystallographic model, decohesion occurs on slip planes [12]. (Courtesy Chapman and Hall.)

(e) Crack extension by shear on alternate planes (either crystallographic or non-crystallographic) [119]. (Courtesy Chapman and Hall.)

less, it is not difficult to envisage specific instances where either crystallographic effects (low stress amplitude and materials with few slip systems particularly in corrosive environments), or fracture of brittle particles ahead of the crack tip (precipitation-hardened alloys having an isotropic arrangement of precipitates) may be the dominant factor in crack growth. It is not surprising that several mechanisms may be valid when one examines the multiplicity of striation profiles that are formed during stage II growth (Fig. 9).

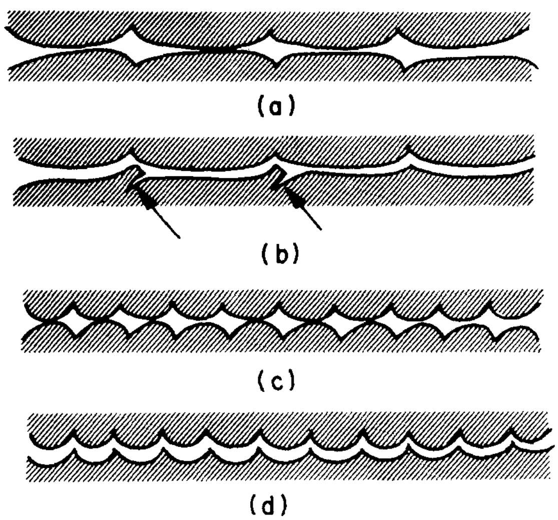


Figure 9 Varieties of ductile striation profile, stress axis vertical, arrows indicate undercutting [139]. (Courtesy American Society for Testing and Materials.)

3.1.2. Transmission electron microscope studies

The experimental difficulties involved in obtaining thin foils from fracture surfaces impaired considerably transmission electron microscope investigations of the dislocation arrangement associated with fatigue cracking, although X-ray analysis [51, 52] had indicated the existence of highly misoriented cell structures. Grosskreutz and Shaw [53] substantiated this for distances between two microns and a few millimetres away from the crack surface in copper, the bulk of the specimen possessing a discontinuous band structure. An extensive study of copper single crystals [9] has revealed both striation profiles and their attendant dislocation distribution. The crack does not appear to follow the sub-grain walls as has been previously suggested [53], and the inter-striation spacing represents several cell diameters (Fig. 10). There is a slight diminution in sub-grain size as growth proceeds. From these observations Klesnil and Lukas [9] conclude that the sub-grain structure is neither a product of attrition of the fracture surfaces nor of deformation at the crack tip during growth in one cycle. Precipitate re-solution or over-ageing may occur in the vicinity of the crack tip [54, 55], but since non-coherent precipitates also dissolve it has been suggested [54] that the processes involve solute diffusion rather than dislocation shearing [56, 57]. At high growth rates and therefore short diffusion times the precipitates remain largely unaffected.

3.2. Quantitative aspects

The main object of investigations into the quantitative aspects of stage II fatigue-crack propagation has been to correlate the crack-growth rate initially with applied stresses and more recently with material properties. Ideally the engineer would like to be able to predict growth rates and hence endurance limits from data provided by simple short tests rather than from a series of full-scale fatigue tests. This ambition has not yet been realized, although the following section will show that substantial progress has been made to this end.

3.2.1. Methods of determining crack propagation rates

Of the many methods available for measuring the speed of crack growth during fatigue, visual observation or photographic recording has been most widely used to date. The technique is both simple and inexpensive, but only provides information concerning the growth rate at the specimen surface, and in thicker specimens where crack tunnelling may occur, the data provided is of questionable value. Growth rates have also been determined from fractographic measurements of the inter-striation spacing, but this method is restricted to fracture surfaces exhibiting well defined striations, and the result is relevant only to the local growth rate rather than to the more important progression of the entire crack front. Reasonable correlation is achieved with data from surface measurements [58]. The most favoured technique at present is the electrical potential or impedance method in which extension of the crack is monitored via the change in potential between points on either side of the crack [59]. Essentially, the area of

Figure 10 Dislocation structure associated with stage II fatigue-fracture surface of copper single crystal section ($1\bar{2}1$). D is electrodeposited material; S is the specimen [9]. (Courtesy Chapman and Hall.)

unbroken specimen is continuously measured, so that errors due to crack tunnelling effects are obviated. The technique is easily automated facilitating crack-growth measurement during long-term fatigue tests, and a resolution of 0.001 in. may be obtained. Other approaches such as acoustic monitoring, continuity strain gauging, eddy current and ultrasonic techniques have also been employed, but not widely accepted on the grounds of expense, resolution or general applicability. The accuracy of crack-growth rate determinations has been considered by Wei [60], who suggests that $\pm$ 40% is realistic for optical measurements. Using the electrical potential technique should yield $\pm$ 30% for low growth rates ($\sim 10^{-7}$ in./cycle) and $\pm$ 10% for higher rates ($\sim 10^{-5}$ in./cycle).

3.2.2. Crack propagation equations

The general philosophy behind prediction of fatigue-crack growth rates has been outlined by McClintock [61], who describes two approaches that may be adopted. First, the damage accumulating in a small region ahead of the crack tip may be computed, and when this exceeds a critical value (which depends on the failure criterion selected), fracture and hence crack growth occurs. For mathematical convenience the analysis considers mode III fracture (antiplane strain), and predicts a growth rate which is proportional to the square of the crack tip plastic zone size. The second method, which is restricted to ductile metals and high stress levels, estimates the extent of the non-cyclic progressive deformation of the crack tip itself,

TABLE I A selection of fatigue-crack growth-rate equations.

Propagation rate $\frac{da}{dN}$	Symbols	Reference
1. $C_1 \frac{\Delta\sigma^2 a}{(\sigma_y - \Delta\sigma)}$	σ_y, yield stress	Head [62]
2. $C_2 \Delta\sigma^3 a$		Frost and Dugdale [63]
3. $Fn\, \Delta\sigma \left[1 + 2\left(\frac{a}{\rho}\right)^{\frac{1}{2}}\right]$	Fn Function ρ, crack-tip radius	McEvily and Illg [64]
4. $C_3 \Delta\sigma^4 a^2 = C_3 \Delta K^4$	ΔK, stress intensity range	Paris and Erdogan [66]
5. $\frac{4\alpha\, \sigma_y}{3\pi\, \mu\, D_c}\left[\frac{\pi\Delta\sigma}{2\sigma_y}\right]^4 . a^2$	D_c, critical displacement for fracture; μ, shear modulus; ν, Poisson's ratio; $\alpha = 1$ for a shear crack, and $1 - \nu$ for a tensile crack	Weertman [72]
6. $\frac{2\,\sigma_g{}^4 a^2}{(\sigma_y + \sigma_u)\, \epsilon_u\, \sigma_u{}^2 E}$	σ_g, gross stress ϵ_u, strain at UTS σ_u E, Modulus	McEvily and Johnston [79]
7. $\frac{\pi^2}{8}\left(\frac{k}{2T}\right)^2 \Delta\epsilon_p{}^{(2\beta+1)} . a .$	k and β constants from cyclic stress-strain curve $\Delta\sigma = k\, \Delta\epsilon_p{}^\beta$ T, approximated to UTS	Tomkins [80]
8. $(\Delta\epsilon\sqrt{a})^5$ net section σ elastic $(\Delta\epsilon\sqrt{a})^2$ net section σ plastic	$\Delta\epsilon$, total strain range	McEvily [84]

and predicts a linear dependence of growth rate upon plastic-zone size.

During the last two decades there has been a multiplicity of equations put forward to describe fatigue-crack growth rate (da/dN) as function of the applied stress levels, crack length and materials properties. Within the scope of this review it would not be possible to even mention them all. Therefore, a short selection (intended to be) representative of the general direction of emphasis is presented in Table I. These will now be discussed briefly.

It was readily established that the cyclic stress amplitude, $\Delta\sigma$, was the dominant stress parameter, although some dispute arose concerning the precise strength of its influence on growth rate [62, 63]. McEvily and Illg [64], using an elastic stress concentration factor approach, challenged the proportionality between growth rate and crack length, and proposed a half power relationship. Paris and co-workers [43, 65, 66] applied the fracture mechanics concept of crack tip stress intensity to fatigue, and the correlation between crack-growth rate and stress intensity amplitude has become most popular in recent years. For a crack length $2a$ the generalized equation may be written,

$$\frac{da}{dN} = C(\Delta K)^m$$

where

$$\Delta K = \Delta\sigma\, \sqrt{a\pi}$$

$\Delta\sigma$ is the gross stress amplitude. A width correction term is omitted for simplicity. Stress intensity is related to the crack tip plastic zone size [67]

$$r_p = \frac{K^2}{4\sqrt{2}\,\pi\,\sigma_y{}^2} \text{ (in plane strain)}$$

$$r_p = \frac{K^2}{2\,\pi\,\sigma_y{}^2} \text{ (in plane stress)}$$

so that, according to the damage approach described previously, the value of m should be 4, while for the crack tip deformation treatment m

should be equal to 2. The interchangeability of K and ΔK is permissible only when the two are equal. Mean stress effects will be considered later. Experimentally the value of m varies between 2 and 7 although some greater values have been noted [68]; 4 is most common. The dominance of the stress intensity range as the controlling factor in cyclic crack growth has been demonstrated for metals [69] and polymers [70] in load shedding experiments, where testing at constant values of ΔK produced constant growth rates. The stress intensity approach should strictly only be applied to regimes which can be described in terms of linear elastic fracture mechanics, i.e. the plastic-zone size at the crack tip should remain small with respect to the crack length and the specimen dimensions. In the literature there has been a tendency, however, for the approach to be universally applied, even to highly ductile materials. While this is obviously in error in terms of absolute values, it has been defended as a useful comparative technique in the absence of a better method. Another fallibility of the stress intensity analysis of cyclic crack growth data involves the calculation of the plastic-zone size in terms of the monotonic yield stress σ_Y. The yield value may radically change under cyclic testing conditions in which higher straining rates and cyclic hardening or softening effects are likely to exist. Also, since it is the stress intensity range which is the major factor in governing the growth rate, the inclusion of the cyclic or repetitive plastic-zone size into the analysis would appear warranted [71]. It is perhaps germane to emphasize at this point, that the stress intensity approach ignores the effects of material properties and the mechanisms involved in crack growth. Nevertheless, despite these criticisms, analysis of fatigue-crack growth data in terms of stress intensity seems justified at present by the correlation with experiment and the absence of a proven superior approach.

Weertman's equation [72] is an example of analyses of fatigue-crack propagation based on an augmented version of the Bilby-Cottrell-Swinden [73] treatment of brittle fracture. It indicates a fourth power dependence on stress intensity but agreement is not unanimous amongst several other similar approaches [74-78]

As stated previously, emphasis has shifted in recent years towards elucidating the effect of material properties on the growth of fatigue cracks. McEvily and Johnston [79] produced from their growth rate equation incorporating the monotonic tensile properties, a "universal" growth curve (Fig. 11). A reasonable agreement was obtained between experiment and prediction, except for Al-Zn-Mg (T6) alloys, in which grain-boundary precipitation effects might have produced inferior resistance to cyclic crack growth. When striations are present on a fracture surface, so providing a value for the growth rate, the "universal curve" may be used in service failure diagnosis, to estimate the magnitude of the operating stresses. Tomkin's equation [80] typifies a slightly more realistic approach by incorporating the cyclic properties (k and β from the cyclic stress-strain curve) of the material. It has subsequently been extended to cover elevated temperature behaviour [81], and high-strength metals in which microfracture in the plastic zone at the crack tip enhances the growth rate relative to that in ductile materials [82]. Good agreement is achieved with experimental data. A similar result is derived by Lehr and Liu [83], who equate

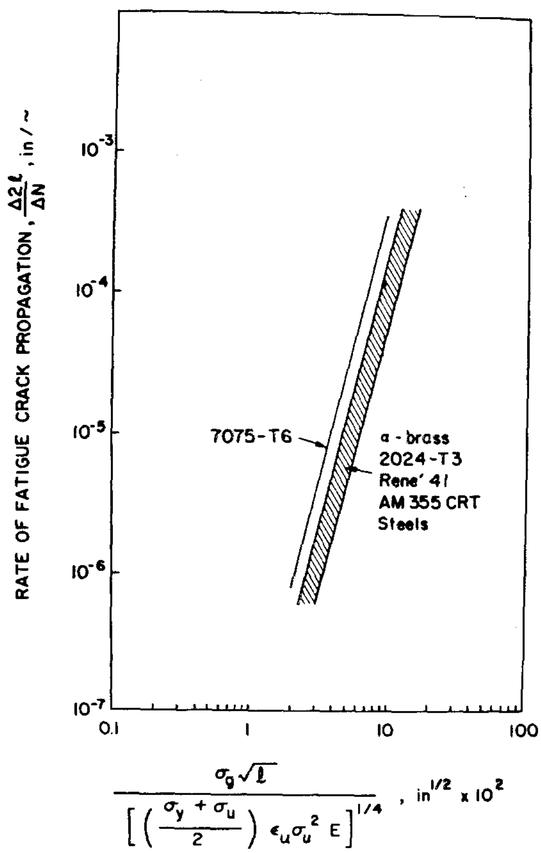


Figure 11 "Universal" fatigue-crack growth-rate curve for several alloys [79]. (Courtesy Wolters-Noordhoff.)

resistance to fatigue-crack growth with cyclic ductility.

The introduction of a new concept of a "strain-intensity factor" [84], permits the comparison of situations at the crack tip which are well outside the range of linear elastic-fracture mechanics. In this approach, crack growth is subdivided into an "elastic stage", in which the net cross-section stresses are below the material yield stress, and a "plastic stage" ($\sigma_{net} > \sigma_y$). The strain intensities in the two ranges are defined as $(\Delta\sigma)/E \sqrt{a}$, and $\Delta\epsilon \sqrt{a}$ respectively; $\Delta\epsilon$ the total strain range, is obtained from the cyclic stress/strain curve of the material in the latter case. For OFHC copper, the only material examined so far, the growth rate was found to be proportional to the fifth power of the strain intensity in the elastic range, and to the second power in the plastic range.

Polymers

Due to the broad spectrum of mechanical response (elastic, linear and non-linear viscoelastic), the application of fracture mechanics and the stress intensity approach to cyclic crack extension in polymers is more limited than in metals. Elastomers and glassy plastics which exhibit bulk elasticity are amenable to such treatment and values for the exponent m in the generalized growth equation

$$\frac{da}{dN} = C(\Delta K)^m$$

are usually similar to those observed for metals [85, 86]. Transparent polymethylmethacrylate has been used to substantiate the predictions of fracture mechanics concerning the growth rate of part-through thickness cracks [85], a vitally important problem in practice.

An alternative to the stress intensity treatment, evolved and described by Andrews [14, 40], involves a "surface work" parameter J. An energy balance criterion for fracture is adopted but J includes the energy dissipated in plastic deformation at the crack tip, mechanical hysteresis and secondary fracture as well as that required for the creation of new surfaces. The value of J, which at fracture is constant, is influenced by stress state, temperature and growth rate (thus permitting cyclic rather than catastrophic propagation). For a viscoelastic material tested at zero minimum constraint the crack-growth rate is given by

$$\frac{da}{dN} = A\nu^{-1} J^{b}$$

where A is a constant and ν the cycling frequency. For materials exhibiting non-viscoelastic behaviour,

$$\frac{da}{dN} = AJ^{b}$$

The value of b varies between 1 and 6, with 2 ± 1 most common, e.g. natural rubber, polyethylene, polymethylmethacrylate and incidentally certain aluminium alloys [14, 40]. For perfect elastic behaviour it is possible to correlate J and K, viz,

$$J = \pi a W = \frac{K^2}{2E}$$

where W is the energy density of the material.

Hence,

$$\frac{da}{dN} = AJ^{b} = A'K^{2b}$$

There is no evidence available at present to justify the more general form of the equations in terms of ΔJ. Although in principle the surface work approach has a greater overall applicability, it is, as yet, based simply on experimental observations. However, the analysis has lately been extended [87] to encompass fatigue-crack growth in viscoelastic materials, and for polyethylene, considerable success has been achieved in predicting fatigue lifetimes, assuming the existence of intrinsic flaws of a similar size to the spherulites.

3.2.3. *General comments on growth-rate equations*

Although for the purposes of fatigue-life prediction some form of linear relationship between growth rate and applied stress is desirable, many of the engineering attempts to achieve linearity over the whole course of crack propagation have perhaps been unrealistic. This is particularly so in sheet and plate materials in which a rotation from "square" to "slant" growth occurs when the plastic-zone size: specimen thickness attains a critical value. McEvily and Johnston [79] point out that the transition to the "slant" mode is associated with a deceleration in growth, and that the overall rate curve tends to be "S" shaped with significantly different gradients on each portion (Fig. 12). A further objection to the single gradient fatigue-crack growth rate curve is provided by the existence of threshold stress intensity below which fatigue cracks do not propagate [88]. Even in circumstances where the square to slant transition is not encountered, it is not uncommon for more than one value of m to be observed in both metals and polymers [89, 14]. This may be due to environmental or microstructural effects, or simply arise from the interaction between the plastic zone and the specimen extremities. It is probable, therefore, that all the equations listed in Table I describe well some regime of the crack propagation period, but it must be emphasized that for generality the whole spectrum of growth rates should be considered. Clearly, for most materials, linear elastic fracture mechanics has insufficient capacity for this.

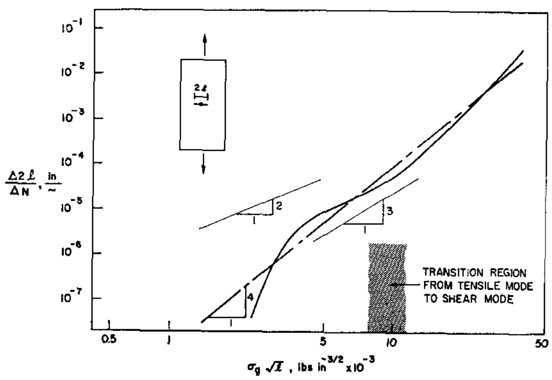


Figure 12 "S" shaped curve indicating true variation of crack-growth rate with stress intensity in an aluminium-zinc-magnesium alloy [79]. (Courtesy Wolters-Noordhoff.)

4. Factors affecting stage II crack propagation

The major factors influencing fatigue-crack growth are the testing variables, environment, temperature and material properties. These are very much inter-dependent variables, and attempting to isolate the effects of each is perhaps an unrealistic exercise. It is probably more logical to classify the outcome of changes produced by the inter-related variables as either cyclic dependent or time dependent, particularly at elevated temperatures. Much of the information found in the literature relates to the influence of the variable on lifetime rather than crack growth. Therefore caution must be exercised in extending these findings to crack propagation. As mentioned previously, many of the definitions of "failure" are purely arbitrary, and it is therefore essential that "failure" occurred after substantial amounts of crack extension. Greatest confidence can be placed in results either from low-cycle fatigue tests or experiments in the high-cycle range using notched specimens. In both instances, stage II crack propagation will account for the majority of the number of cycles to failure.

4.1. Testing parameters

So far, only the simplest cases of fatigue have been considered, i.e. fully reversed cycling with zero mean stress, and undirectional cycling where the values of stress amplitude and maximum are identical. Crack propagation in real situations is more complex, and the influence on the basic equations discussed previously, of mean stress, cyclic frequency, and mode of testing will now be considered.

4.1.1. Mean stress

In most cases fatigue involves stress fluctuations about a non-zero stationary stress, the magnitude of which would be expected to alter the extension rate of a growing crack. Generally, an increase in mean stress, σ_m, leads to an increase in growth rate [90-93], but instances have been reported where there is no measurable effect [94-96]. Relaxation at the crack tip has been put forward as a possible explanation for the latter [92]. It is more usual to refer to the stress ratio, R ($\sigma_{\min}:\sigma_{\max}$), and for a constant stress intensity range the cyclic crack growth rate increases as the value of R increases greater than zero (Fig. 13). Changes in negative values of R have little effect on $\mathrm{d}a/\mathrm{d}N$ [97], indicating the minor influence of compression loading on propagation. From studies of aluminium alloys Broek and Schijve [92] have derived a growth-rate equation incorporating mean stress effects.

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C_1\left(\frac{\Delta K}{1-R}\right)^m \exp\left(-\,C_2R\right)$$

Its limitations at high propagation rates have been exposed by Hudson and Scardina [97], who favour the equation due to Foreman *et al* [98], i.e.

$$\frac{\mathrm{d}a}{\mathrm{d}N} = \frac{C(\Delta K)^m}{(1-R)K_c - \Delta K}$$

K_c is the critical stress intensity for fracture under plane stress conditions. The overall validity of this equation will be mentioned later. For thicker specimens Pearson's equation [93] describes the

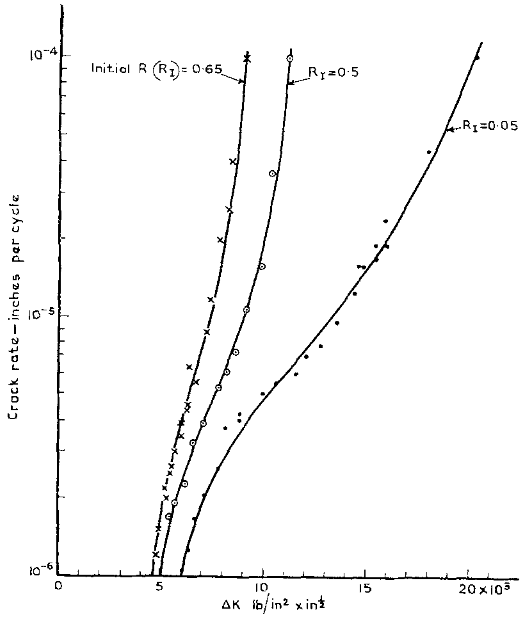


Figure 13 Effect of mean stress on fatigue-crack propagation in aluminium alloy RR58-fully heat treated [92]. (Courtesy Ministry of Technology.)

influence of mean stress in terms of the plane strain fracture toughness K_{IC},

$$\frac{da}{dN} = \frac{C(\Delta K)^m}{[(1 - R)K_{IC} - \Delta K]^{\frac{1}{2}}}$$

For polymethylmethacrylate, a growth rate equation which includes both mean stress and frequency terms, has been deduced from a multiple regression analysis of experimental data by Mukherjee and Burns [86].

$$\frac{da}{dN} = C(f)^{\beta_1} (\Delta K)^{\beta_2} (K_{\text{mean}})^{\beta_3}$$

The values obtained for β_1, β_2 and β_3 were $-$ 0.43, 2.39, and 2.13 respectively, which is indicative of a strong influence of mean stress for this material, and is probably typical of polymers. It was found that neither frequency nor mean stress altered the exponent β_2 (or m) of the basic propagation equation. Natural rubber is exceptional in that a mean stress produces an increase in total fatigue life provided that the minimum strain per cycle does not fall to zero [14].

4.1.2. Frequency

The influence of test frequency arises from two sources; (a) environmental effects, and (b) strain-rate effects. Both are peculiar to the material under examination, although most information at hand fails to distinguish between them. At normal frequencies the rate of crack propagation in metals increases as the cyclic frequency is reduced [99] (Fig. 14) although in the ultrasonic range ($\sim$ 10000 Hz) there is some evidence that this tendency may be reversed [100, 101]. This may be due to a heating effect producing structural degradation as is the case in polymers at much lower frequencies [14]. The importance of straining rate, rather than frequency has been emphasized by Miller [102, 103], who demonstrates the existence of critical strain rate (55×10^{-4} sec^{-1} at room temperature for aluminium), above which there is no further effect on lifetime in high strain torsion fatigue. Although this result cannot be directly equated with the propagation of a crack well removed from its initiation stage, it is reasonable that there should be a critical straining rate above which time dependent effects would become negligible. Information from square wave cycling tests, in which theoretically, the strain rate is infinite, should unequivocally reveal the influence of cyclic frequency. The stress wave-form (sinusoidal, square, sawtooth, etc) may also influence crack growth to a secondary extent particularly at elevated temperatures, and not unexpectedly square wave cycling appears to be the most deleterious [104].

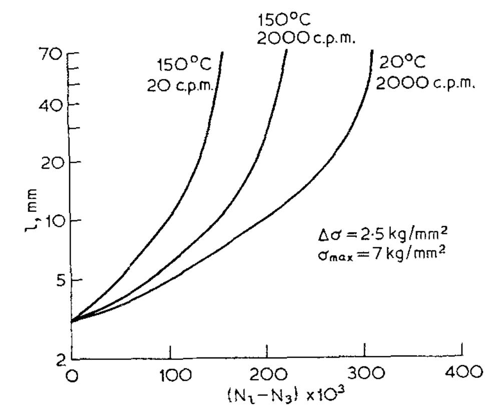


Figure 14 Effect of frequency and temperature on fatigue-crack propagation in a clad aluminium-copper alloy [98]. (Courtesy N.L.R. Amsterdam.)

4.1.3. Mode of testing

The majority of service components are subjected not to uniaxial, but to multiaxial stresses/strains. It is therefore pertinent to enquire how useful is the wealth of data on uniaxial fatigue-crack

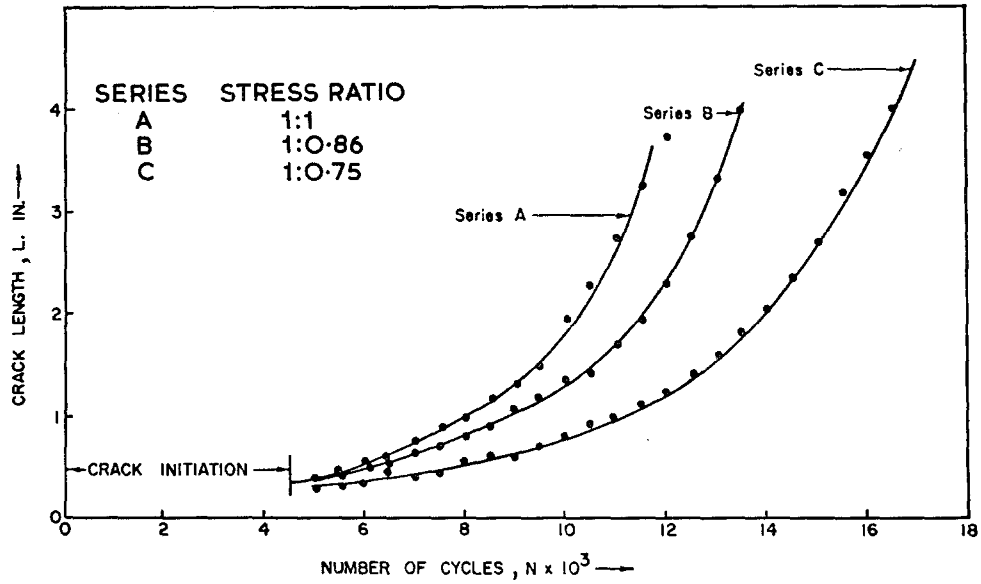


Figure 15 Influence of stress ratio on crack growth during biaxial testing of an aluminium-copper alloy [105]. (Courtesy Society for Experimental Stress Analysis.)

propagation in assessing the durability of materials in real situations. The present paucity of information on cyclic crack extension under multiaxial conditions renders the question almost unanswerable, but the indications are that uniaxial data are of little quantitative assistance to the design engineer. Christensen and Harmon [58] have pointed out the importance of stress ratio and material anisotropy during biaxial fatigue, and indicate that the crack-growth rate may be increased by a factor as large as three over that measured in uniaxial cycling. Both similar and conflicting results have been quoted by Hunt *et al* [105], who find that a change in the principal stress ratio from one to two increases the "crack life" to more than that for uniaxial testing (Fig. 15). In all cases they found that biaxial fatigue at elevated temperatures led to an increase in life, which conflicts with most data on uniaxial testing. Recently Joshi and Schewchuk [106] examined the influence of a stress parallel to a crack plane, and contrary to the predictions of linear elastic fracture mechanics [107], find a change in growth rate as the stress ratio is altered. A linear plot of log. growth rate versus equivalent stress intensity factor is achieved, the equivalent stress being defined as

$$\sigma_1^2 - \sigma_2^2 - \mu\sigma_1\sigma_2$$

where μ is Poisson's ratio. They also predict a diminution in rate with respect to uniaxial cycling on applying a second stress (smaller than σ_1), by virtue of the reduction in crack tip plastic-zone size, but unfortunately they do not attempt an experimental verification. Clearly, a great deal of work is necessary before crack propagation under multiaxial conditions can be fully understood.

4.2. Environment

Very few generalizations may be made concerning the influence of environment on fatigue-crack propagation since the precise reactions taking place at the crack tip, which produces the effects, are still largely a matter of conjecture. Until fairly recently, only a welter of random data was available which pertained to specific material-environment systems. A few representative findings will be presented in this section. Corrosion fatigue in aluminium alloys, titanium alloys and steels has been well summarized by Wei [60], and Ford [108] has considered aluminium alloys in depth.

The presence of a vacuum is usually beneficial in reducing the rate of cyclic crack growth (Fig. 16), although the improvement may cease at a critical pressure [109]. This again is "frequency" sensitive, the important factor being the ability to maintain the region adjacent to the crack tip atomically clean during each cycle, so

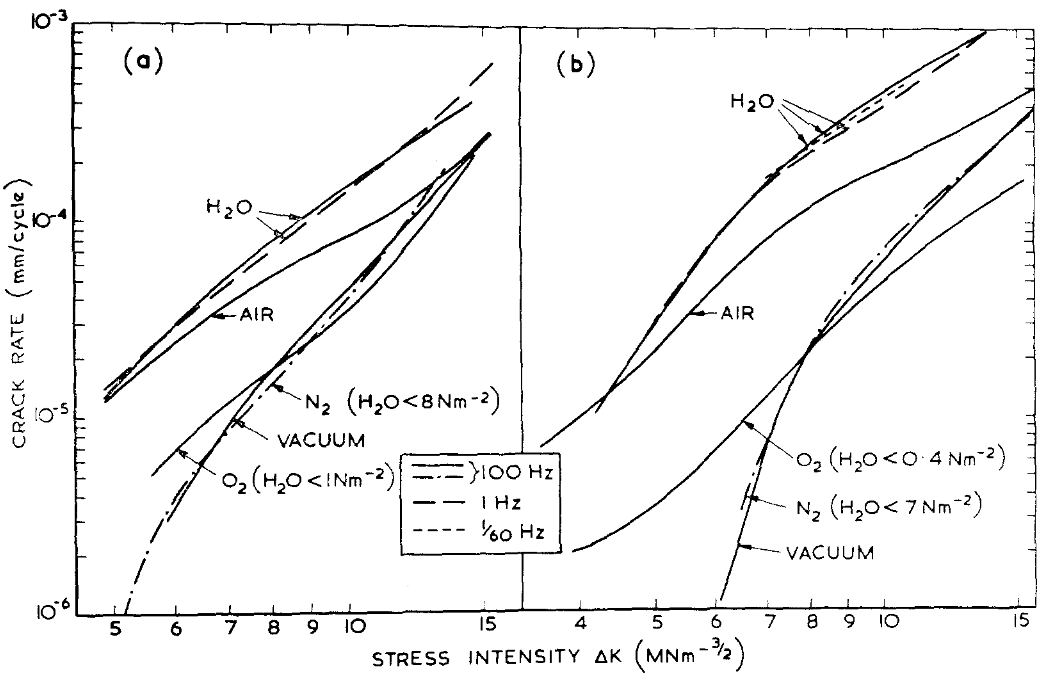


Figure 16 Fatigue-crack growth in (a) 5070A aluminium alloy and (b) 683 aluminium alloy, in various environments [115]. (Courtesy Wolters-Noordhoff.)

that, it is suggested [110], a degree of re-welding may occur and reduce the incremental growth. Considerable controversy exists concerning the concept of re-welding and the dispute is well discussed by Achter [111]. The benefit of vacuum may sometimes not be apparent at elevated temperatures when bulk oxidation occurs and produces a hardening effect [112]. For copper, a vacuum of 7×10^{-6} torr produces an increase in total life over that in air by a factor of twenty [113], and in this case, oxygen is the effective constituent. In low alloy steels, and certain copper, titanium, magnesium, aluminium and zirconium alloys the water vapour content of the environment is the controlling factor [114-117]. It is thought that a crack tip reaction involving hydrogen occurs, in which pressure build in voids ahead of the tip causes embrittlement [118]. There are also data indicating that fatigue-crack propagation rates in moist hydrogen are approximately a third of those in dry hydrogen [119]. Pelloux [120] states that in aluminium alloys the crack tip reaction is electrochemical in nature, but dismisses embrittlement or dissolution in favour of a "stress sorption" process in which the metal bonds at the crack tip are weakened by adsorption of the environment. There is usually a concentration (or pressure) range of active constituent which affects crack growth; the extremes are frequently sensitive and are probably indicative of saturation or denudation of the crack tip region [109, 115].

Environmental effects are most apparent during the early periods of stage II propagation (low values of $(da)/dN$ [88]), and when plane strain conditions exist at the crack tip [121, 122]. In aluminium alloys, however, environmentally induced changes tend to persist for all growth rates in thick specimens [60]. The rate controlling process for $(da)/dN < 10^{-4}$ in./cycle is alleged to be the creation of new surfaces rather than transport of the environment to the crack tip or diffusion to the material ahead of the crack [60], but at higher growth rates this situation may alter [123]. Corrosion fatigue-crack growth in an aluminium alloy–water system has been demonstrated to be a single thermally activated process over a small range of temperatures (22 to 107° C). Wei [121] derived a growth-rate equation from these findings

$$\frac{da}{dN} = AF(\Delta K)\exp\left[= \frac{u(\Delta K)}{RT}\right]$$

where A is a constant, $F(\Delta K)$ is the crack driving force, and $u(\Delta K)$ an apparent activation energy. Unfortunately the accuracy of the activation energy computation precluded an estimate of the crack driving force. Hartman and Schijve [124] examined the growth-rate equation of Foreman *et al* [98] incorporating mean stress effects, and found that, in general, for aluminium alloys in several environments the equation was not applicable. However, by incorporating a threshold stress intensity term, ΔK_0, deduced from measurements in argon, much better agreement was achieved.

The beginnings of a rationale for fatigue-crack propagation under corrosive environments seem to be emerging from some of the latest work. Meyn [125] has classified two varieties of corrosion fatigue according to the relative values of the maximum stress intensity at the crack tip, K_{max}, and the critical stress intensity for stress corrosion cracking, K_{ISCC}. In type A ($K_{max} > K_{ISCC}$), stress corrosion cracking takes place during each cycle and is highly frequency sensitive, but less so when $K_{mean} > K_{ISCC}$. This form of corrosion fatigue has been observed in steels [126] and titanium alloys [127], and is usually restricted to high-strength alloys. Type B corrosion fatigue ($K_{max} < K_{ISCC}$) does not depend upon environmental cracking in each cycle, and is most effective at low stress levels [121, 128]. The crack-growth rate is virtually independent of frequency, and for titanium alloys Meyn [125] attributes the damage to either hydrogen embrittlement, or adsorption of some species which gives rise to cracking in a similar manner to liquid metal cracking (cf. Pelloux for aluminium alloys). Barsom [129] has considered type B corrosion fatigue-crack growth in aqueous media, and lists three reactions that may occur at the crack tip; (a) reduction in surface energy, (b) hydrogen embrittlement, (c) metal dissolution or active path corrosion. The first is usually restricted to brittle materials. It is possible to distinguish between active path corrosion and hydrogen embrittlement in thin specimens by applying either anodic or cathodic currents to the specimen [130, 131]. Hydrogen embrittlement is indicated if a small cathodic current produces a decrease in life and an anodic current prolongs life. In thick specimens, however, closed circuits are set up in localized regions and the applied potential method is inadequate for isolating the fracture mechanism [129]. The most recent studies on type B corrosion fatigue [132] are particularly significant since they indicate that environmentally accelerated effects occur only during the loading period of the stress cycle. For a 12 Ni-5Cr-3Mo maraging steel, a strain rate insensitive material, it was demonstrated that while sinusoidal, triangular and positive-sawtooth load cycling in a corrosive medium produced accelerated crack-growth rates with respect to those measured in air, square wave and negative-sawtooth load cycling had no effect. More work is required to establish the generality of this observation.

The field of fatigue-crack propagation in polymeric materials in aggressive environments is largely unexplored. However, since static stress corrosion occurs in many types of polymer (polyethylene in water and surface active liquids, rubber in ozone containing atmospheres, and polymethylmethacrylate in organic solvents) [40], it is to be expected that cyclic crack extension will be affected also.

4.3. Temperature

Cyclic crack growth at elevated temperatures has been a somewhat neglected sector of fatigue (as exemplified by a conference in 1968 entitled "Fatigue at High Temperature" [133] in which not a single result on crack growth was presented). However, the need for such information is rapidly becoming apparent, particularly to those engaged in the generation of more realistic design criteria. For most metallic materials, the situation at high temperatures ($T > 0.5T_m$ K), is complex, since time dependent effects become of prime importance as a consequence of both the degradation in mechanical properties and the acceleration of environmental influences. Fatigue-crack propagation becomes more rapid [99, 134, 135] (Fig. 17), and the effects of frequency (or strain rate) and environment are accentuated. [135-137]. For example, in FV535 steel the crack propagation rate at 500° C cycling at 10 cpm is 137% greater than that at 20° C, and a change in frequency from 10 to 1500 cpm produces a reduction of 11% in growth rate at 20° C, but at 500° C a 34% fall occurs [135]. Such effects tend to diminish at high growth rates or for high values of the stress intensity range. In some steels, the exponent in the ΔK v (da/dN) plot decreases, but in others it remains unchanged during constant frequency testing [136]. Schijve and De Rijk [99] have attempted to isolate the contributions to fatigue-crack growth arising from time and cyclic dependent processes. They found that the rate of extension of a fatigue crack under constant load was about two orders of magnitude less than when under cyclic loading, and concluded that, since creep strain was not all concentrated at the crack tip, the combined effects of fatigue and creep were less than additive. The creep contribution to fatigue cracking in polymethylmethacrylate has been estimated to be less than 10% [70]. Tests measuring the influence of temperature on total fatigue life indicate similar results, but initiation is often located at voids created by creep reactions [138, 139]. A recent examination [140] of the effect of temperature (up to $0.5T_m$) on growth rates under low stress levels in stainless steels indicates that more than one process is dominant over the temperature range (in conflict with Wei's finding for aluminium alloys in water over a smaller temperature range). Considerable work remains to be done in elucidating the nature and extent of the damaging processes (fatigue or creep) at various temperatures before this aspect of fatigue-crack propagation can be fully understood or confidently accounted for by the designer.

4.4. Material properties

The influence of structure on fatigue-crack

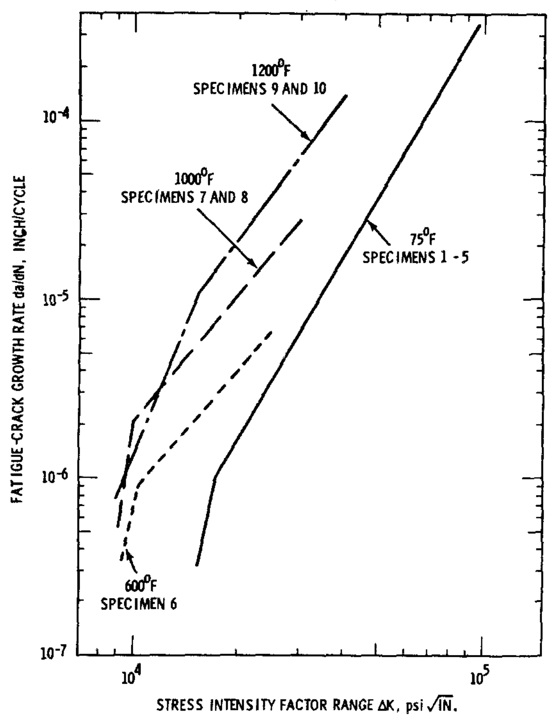


Figure 17 Fatigue-crack propagation rates in type 304 stainless steel at various temperatures [138]. (Courtesy American Society for Metals.)

propagation in metals has been considered at length by Laird [141]. In general terms, it may be stated that structural effects become increasingly important as the cyclic stress (strain) amplitude diminishes, and that specific features will exert an influence only if their dimensions are of the same order as the local increment of growth per cycle. Thus, in high strain fatigue, cold work has little effect on crack growth, since the initial cycles create a degree of plastic deformation comparable with that existing in the material. Grain size, also, appears unimportant particularly in high stacking-fault energy materials which will contain dislocation sub-grains. At low stresses and long fatigue lives, cold worked materials soften, although not generally to the annealed state, and a growth rate lower than that of the annealed state results. The enhanced work hardening capacity of low stacking-fault energy materials also retards cyclic crack growth. Grain-size influences become apparent in the absence of a dislocation cell structure; a large grain diameter being less resistant to crack propagation.

Orientation effects in sheet material have been extensively investigated and again the results are conflicting. Schijve [142, 143] found that a higher resistance to fatigue-crack growth existed along the transverse plane to the rolling direction, whereas Rooke *et al* [144] noted the opposite in a similar alloy. Orientation effects can arise from two sources; a crystallographic texturing and a fibred effect produced by alignment of second phase particles, inclusions, etc. Much of the previous data does not distinguish between these. However, Weber [145], has deduced that crystallographic influences are negligible in β brass, and that in 305 steel the measured orientation effect, an acceleration in growth in the transverse direction, may be attributed to the mechanical changes produced by the alignment of inclusions. For aluminium, copper, and an Al-Mg alloy, Le May and Nair [146] found that the fatigue properties in the transverse direction were superior to those along the plane parallel to the rolling direction, and contrary to Weber they conclude that this was due to crystallographic texturing and that inclusion alignment has little effect. A more systematic approach is required in order to resolve the dispute surrounding this phenomenon.

There has been a similar amount of conflicting data reported concerning the effect of second phase particles on fatigue-crack propagation,

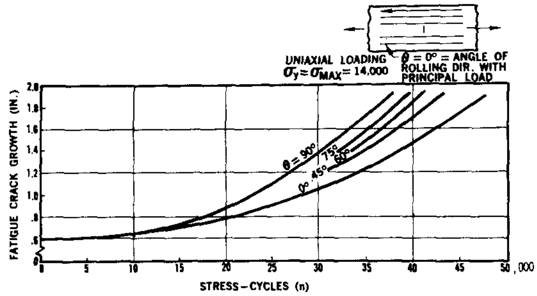


Figure 18 The effect of anisotropy on fatigue-crack growth [58]. (Courtesy American Society for Testing and Materials.)

although the rationale for this now seems to have been established. Inclusions or second phase particles have a marked effect on overall fatigue life, which for steels has been shown to be inversely proportional to the inclusion size [147, 148]. However, it is the initiation stage that is greatly accelerated, particularly by the larger inclusions (> 1 μm for aluminium alloys) [4]. At low rates of fatigue-crack growth, the effect of particles is very small, and apart from a local increase in growth rate in the region adjacent to the particle the macroscopic propagation rate is unaltered. At high growth rates, when particle cleavage rather than loss of cohesion with the matrix may occur, void formation ahead of the crack tip and subsequent ductile tearing may induce an increase in propagation rate [149]. The cumulative effect, naturally, is controlled by the density and size of the particles. In Al-Cu-Mg alloys Broek and Bowles [150] have found that the crack growth rate is proportional to the GP II zone size in the range 200 to 3000Å.

In comparison with metallic materials, present knowledge concerning the structure of polymers is extremely limited. Most important polymers are made up of "crystalline" and amorphous regions each of which influences mechanical behaviour. The degree of crystallinity may often be increased by thermal treatment or the application of strain, and in general, static strengths are highest in fully crystalline, highly oriented polymers. There is some evidence that fatigue properties also follow this trend [151], but again the field is largely unexplored. Hertzberg *et al* [39] have equated the fatigue-crack growth behaviour of several polymers with their damping characteristics, and state that best resistance to crack propagation is achieved when substantial internal energy dissipation mechanisms exist. Nylon 66 which had a broad energy absorption spectrum exhibited the slowest growth rates. The fatigue properties of two phase (reinforced) polymers are usually poor [40].

4.4.1. Fracture toughness, crack opening displacement and fatigue-crack propagation

As stated earlier, one of the aims of the engineer is to be able to predict fatigue-crack propagation rates from data obtained from tests less complicated than actual measurement. Consequently there have been several attempts to correlate cyclic crack growth with fracture toughness in "brittle" materials and crack opening displacement in "ductile" materials. Based on an empirical relationship between plane strain fracture toughness, K_{IC} and the plastic flow property of a material, and assuming a fourth power dependence of da/dN on stress intensity, Krafft [152] proposed that

$$\frac{da}{dN} = \frac{C(\Delta K)^2 (K_{max})^2}{K_{IC}^2}$$

or

$$\frac{C(\Delta K)^4}{K_{IC}^2}$$

for zero to tensile cycling. However, the extension of this basic model for plane strain instability [153] to fatigue has been questioned [154]. From observations of crack growth in aluminium alloys Pearson [93] obtained best fit with experimental data using,

$$\frac{da}{dN} = \frac{C(\Delta K)^m}{[(1 - R)K_{IC} - \Delta K]^{\frac{1}{2}}}$$

where R is the stress ratio. For da/dN in in./cycle the values of C and m were 3.3×10^{-20} and 3 respectively. Aluminium-zinc-magnesium alloys were exceptional in that the equation seriously underestimated the growth rate at high values of R [155]. In cases where extremely thick specimens are necessary to obtain the value of K_{IC} in fracture toughness testing, Pearson recommends the fatigue-crack growth-rate determination as an alternative. For thin specimens when plane stress conditions predominate, the cyclic crack propagation rate has been derived by Foreman *et al* [98],

$$\frac{da}{dN} = \frac{C(\Delta K)^m}{(1 - R)K_c - \Delta K}$$

where K_c is the critical stress intensity for fracture under plane stress. Over a relatively small range of growth rates good agreement was achieved for two aluminium alloys in air. The general applicability of the equation has been mentioned previously. Studies on high strength steels [156, 157] indicate that the correlation between growth rate and fracture toughness is also a function of environment, with no obvious relationship existing for tests carried out in inert atmospheres. Steels with the lowest K_{IC} values were most sensitive to environment.

Under elastic conditions the separation between the two surfaces of a stationary crack, or the "crack opening displacement", δ, may be related to the stress intensity at the crack tip [158],

$$\delta = \frac{4K^2}{\sigma_Y E}$$

The crack opening displacement is equivalent to the product of the plastic zone size and the yield strain, so that during cyclic deformation, considering the reversed plastic zone size [159],

$$\delta = \frac{(\Delta K)^2}{2\pi\,(2\sigma_Y)\,E}$$

Therefore the "damage accumulation" approach to fatigue-crack growth would predict that da/dN is proportional to δ^2 while the "progressive deformation of the crack tip" treatment suggests a direct proportionality. Examination of a simple model involving decohesion on planes of maximum shear indicates that [50]

$$\frac{da}{dN} = \tfrac{1}{2}\delta$$

Of the small amount of experimental evidence available both the linear dependence [160, 161], and the second power dependence [120, 159] of growth rate on crack opening displacement have some support. For several alloys it was shown that the value of δ exceeded that of da/dN at growth rates greater than 5×10^{-5} in./cycle [120], although a subsequent comparison of adjacent-striation-spacing-ratios and their associated δ values indicated that direct proportionality existed only at "larger loads" (i.e. $da/dN > 4 \times 10^{-4}$ in./cycle) [159]. Donahue *et al* [162], however, assert that the crack opening displacement treatment should be restricted to square growth, which for steels involves $da/dN < 10^{-4}$ in./cycle. It is clear that considerably more work is required before this problem is resolved. It is possible that the occurrence of secondary fracture at high growth rates, in addition to sliding-off, may provide part of the answer.

5. Future trends

The preceding sections have shown, that during the last few years, substantial progress has been made both in understanding and designing for crack propagation under cyclic loading. However, it cannot be said that the problem has been completely overcome in any of its aspects, and in many areas there is still a pressing demand for raw experimental data. Attempts at formulating generalized expressions to describe fatigue-crack growth under a wide variety of conditions have met with only limited success. The following paragraphs will suggest possible future developments in the study of cyclic crack propagation.

It is likely that the general form of the fatigue test will become more complex in an effort to reproduce real engineering situations. For instance, fatigue-crack growth under random or multiaxial stressing may be investigated in specimens containing keyways, notches, corners, large changes in section, holes, welds, etc. As was pointed out recently [1], the very early period of stage II growth is probably most significant, and attention may be turned on this region. If so, environmental effects would become critical, as would be the magnitude of the threshold stress intensity necessary for crack growth. During this early period, where growth rates are low ($\sim 10^{-7}$ in./cycle), it might be expected that propagation would become more structure-sensitive, and thus allow more scope for the metallurgist to design alloys with improved fatigue resistance. A refinement in crack measuring techniques would also be necessary.

It does not now appear likely that the "elixir" of a single straight line to describe the whole course of stage II propagation will be found within the near future. Therefore, according to his individual tolerances, the designer must decide whether to accept the generalized growth-rate equation using a suitable value for m, or to consider each sub-division in the growth rate curve separately. A threshold stress (strain) term should be incorporated in future growth rate expressions; the magnitude of this may be regarded as a measure of the corrosive power of any particular environment [162]. It would also be advantageous if there was general acceptance of the definitions of "initiation" and "failure". The correlation between fracture toughness, or crack-opening displacement and fatigue-crack propagation is still in its infancy, and much remains to be done. On the philosophy that intrinsic defects will always exist in materials, it is important that not only the critical flaw size be known, but also the period required for existing flaws to attain that size.

Fatigue-crack growth at elevated temperatures presents intriguing problems to both the materials scientist and the engineer. The task of the former is to ascertain the dominant mechanism (creep or fatigue) causing structural degradation, and specify alloys accordingly. Dynamic experiments carried out in high voltage transmission electron microscopes should provide useful information to this end. When the present shortage of experimental data is over, the engineer will have to generate predictive expressions for the lifetimes of components subjected to both cyclic and time dependent deformation. This might prove very difficult, since long term life predictions under static loading alone are not particularly reliable at present. Crack-growth tests which might be performed would probably have loading spectra incorporating hold or dwell periods (either on or off-load). It is to be hoped that the chemical aspects of environmental influences will be pursued more successfully than has been the case at room temperature.

Knowledge of the structure-property relationship in polymers is limited in comparison with that for metals and alloys. As commercial polymeric materials become increasingly important, so the demand for information concerning their mechanical behaviour will grow. The developments in fatigue-crack propagation studies will probably follow those of metals, with the behaviour of viscoelastic polymers being roughly equated with metallic response at elevated temperatures. Cyclic crack growth in reinforced polymers should attract considerable attention.

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